NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS

2016 HSC Course Assessment Task 3
(Trial Examination) June 21, 2016

General Instructions

- Working time –3 hours
  (plus 5 minutes reading time).
- Write using blue or black pen.
  Diagrams may be sketched in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt all questions.

Section I - 10 marks

- Mark your answers on the answer sheet provided.

Section II – 90 marks

- Commence each new question on a new page.
- Show all necessary working in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: ……………………………………………

 # BOOKLETS USED: ……….

Class (please ✓)

☐ Mr Hwang

☐ Mr Zuber

☐ Mr Berry

☐ Ms Lee

☐ Ms Ziaziaris

Question	1-10	11	12	13	14	15	16	**Total**
Marks	$\overline{10}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{15}$	$\overline{100}$

Section 1: Multiple Choice– 1 mark each.

Q1. The exact value of $\operatorname{cosec}\frac{7\pi}{6}$ is

(A) -2

(B) $-\frac{2}{\sqrt{3}}$

(C) $\frac{2}{\sqrt{3}}$

(D) 2

Q2. The value of

$$\sum_{i=10}^{21} 2i - 30$$

is

(A) 8

(B) 10

(C) 11

(D) 12

Q3. Which line is perpendicular to the line $3x + 4y + 7 = 0$?

(A) $4x + 3y - 7 = 0$

(B) $3x - 4y + 7 = 0$

(C) $8x - 6y - 7 = 0$

(D) $4x - 7y + 7 = 0$

Question 4 on the next page

Q4. Using the trapezoidal rule with 4 subintervals, which expression gives the approximate area under the curve $y = x\log_e x$ between $x = 1$ and $x = 3$

(A) $\frac{1}{4}(\log_e 1 + 6\log_e 1.5 + 4\log_e 2 + 10\log_e 2.5 + 3\log_e 3)$

(B) $\frac{1}{4}(\log_e 1 + 3\log_e 1.5 + 4\log_e 2 + 5\log_e 2.5 + 3\log_e 3)$

(C) $\frac{1}{2}(\log_e 1 + 3\log_e 1.5 + 4\log_e 2 + 5\log_e 2.5 + 3\log_e 3)$

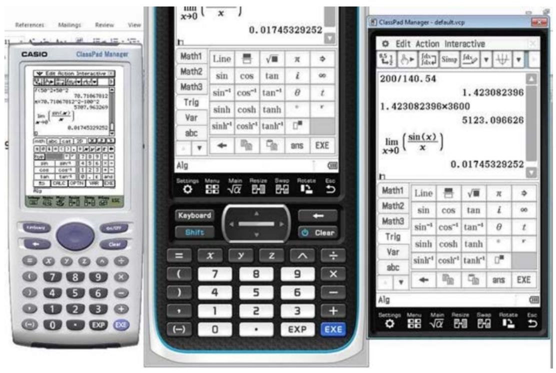
(D) $\frac{1}{2}(\log_e 1 + 6\log_e 1.5 + 4\log_e 2 + 10\log_e 2.5 + 3\log_e 3)$

Q5. A student (not in NSW) is using technology in their exam to calculate a limit.

Their calculator tells them that

$$\lim_{x\to 0}\frac{\sin x}{x} = 0.017453\ ...$$

What happened?



(A) The calculator made a rounding error.

(B) The calculator is in degrees.

(C) The calculator is in radians.

(D) The calculator is in gradiens.

Question 6 on the next page

Q6. A population of sea monkeys is observed to fluctuate according to the equation

$$\frac{dP}{dt} = 40\sin(0.1t),$$

where P is the sea monkey population and t is the time in days.

During which day does the population first start to decrease?

(A) Day 15

(B) Day 16

(C) Day 31

(D) Day 32

Q7. The solution to $3x^2 + 7x > 6$ is

(A) $-\frac{1}{2} < x < -\frac{1}{3}$

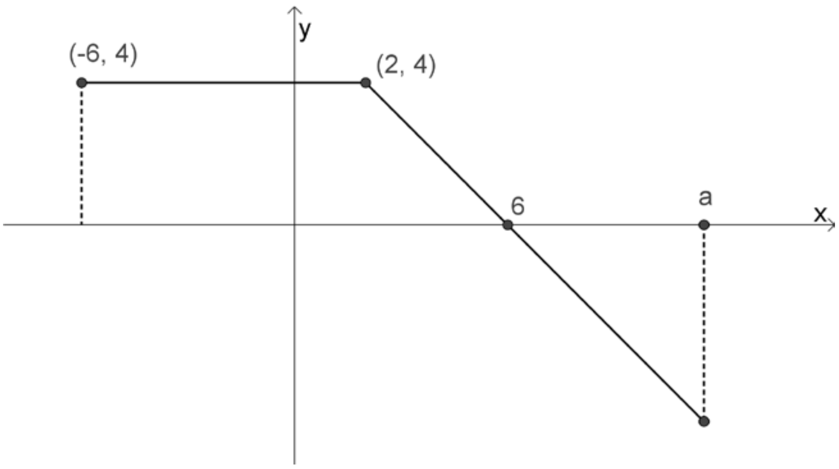
(B) $x < -\frac{1}{2}, x > -\frac{1}{3}$

(C) $x < -3, x > \frac{2}{3}$

(D) $-3 < x < \frac{2}{3}$

Question 8 on the next page

Q8. Using the graph of $y = f(x)$ below,



NOT TO SCALE

determine the value of a which satisfies the condition:

$$\int_{-6}^{a} f(x)dx = 8$$

(A) 8

(B) 10

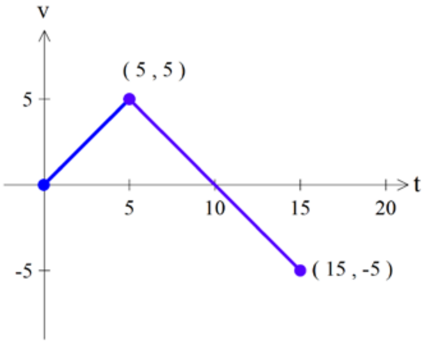
(C) 12

(D) 14

Question 9 on the next page

Q9. A particle is moving along the x-axis.

The graph shows its velocity v metres per second at time t seconds.



When $t = 0$, the displacement x is equal to 5 metres.

What is the maximum value of the displacement x?

(A) 12.5 m

(B) 25.0 m

(C) 30.0 m

(D) 47.5 m

Q10. The derivative of $y = x^x$ is

(A) $x \cdot x^{x-1}$

(B) $(1 + \log_e x) \cdot x^x$

(C) $2x \cdot x^x$

(D) $(x \log_e x) \cdot x^x$

End of Section I

Section II – Short Answer 90 marks

Question 11 (15 marks) Commence on a NEW page. Marks

(a) Rationalise the denominator $\dfrac{1-\sqrt{3}}{5-\sqrt{3}}$ 2

(b) Derive the value $\lim\limits_{x\to 3} \dfrac{x^3-27}{x^2-9}$ 2

(c) Solve $|\,2x+1\,| > 6$ 2

(d) Sketch the region defined by the intersection of the inequalities: 3

$$y \geq (x-1)^3$$

$$y \leq \sqrt{1-x^2}$$

(e) Differentiate $y = 4x^3 - \sqrt{x}$ 2

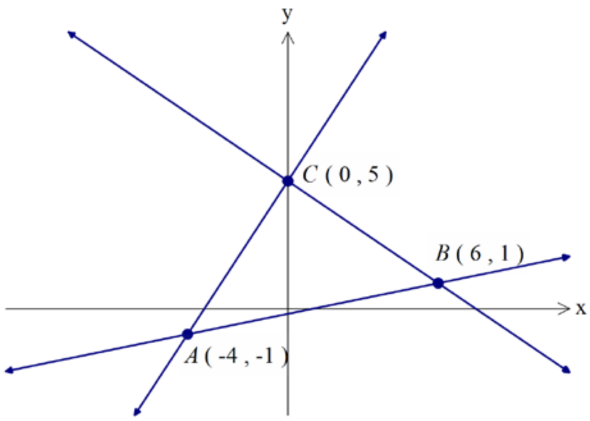
(f) Find $\displaystyle\int \left(x^3 - \frac{2}{x}\right)\, dx$ 2

(g) Evaluate $\displaystyle\int_0^{\pi} \sin 2t \;\, dt$ 2

End of Question 11

Question 12 (15 marks) Commence on a NEW page. Marks

(a) The points $A(-4,-1)$, $B(6,1)$, $C(0,5)$ are defined in the Cartesian plane.



i) Show that the line passing through points A and B is $x - 5y - 1 = 0$ 1

ii) Find the distance AB. 1

iii) Find the area of the triangle ΔABC. 3

(b) Differentiate:

i) $y = \cos(7 - x^4)$ 2

ii) $y = \log_e \dfrac{2x+1}{x-1}$ 2

(c) Find:

i) $\int 3e^{-5x}\, dx$ 2

ii) $\int x^2(1 - \sqrt{x})\, dx$ 2

iii) $\int \dfrac{6x}{x^2 - 1}\, dx$ 2

End of Question 12

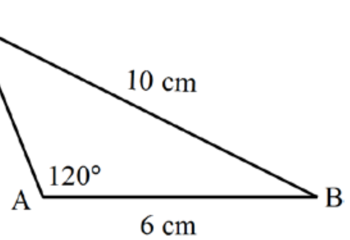
Question 13 (15 marks) Commence on a NEW page. Marks

(a) Sketch the parabola 4

$$y^2 + 8x - 2y + 25 = 0$$

clearly showing the location of the vertex, the focus point and the directrix.

(b) The diagram shows triangle ABC with sides AB = 6 cm, BC = 10 cm, and $\angle CAB = 120°$. 4



Find the exact value of $\tan C$.

(c) For what values of k does the line $y = 5x + k$ intersect with the curve $y = x^2 + 3$? 3

(d) Solve the equation $4\sin^2 x + 6\,\text{cosec}^2 x = 11, \ \ 0 \le x \le 2\pi$. 4

End of Question 13

Question 14 (15 marks) Commence on a NEW page. Marks

(a) Given the function $f(x) = 6x^3 - x^4$

i) Find the coordinates of the points where the curve crosses the axes 1

ii) Find the coordinates of the stationary points and determine their nature 4

iii) Find the coordinates of the points of inflexion 2

iv) Sketch the graph of $y = f(x)$, clearly indicating the intercepts, stationary points and points of inflexion. 3

(b) In the BBC television documentary "Inside the Factory", a production manager described the process of manufacturing bulk quantities of baker's yeast.

He stated that, "the yeast doubles every 3 hrs" and that, "it takes 2½ days to fill the 30,000 kg capacity fermentation vat".

The growth of the yeast is modelled using the equation,

$$P = P_0\, e^{kt}$$

where P is the mass of the yeast in kilograms at time t in hours, and P_0 is the initial amount of yeast put into the fermenter.

i) Find the exact value of k that produces a doubling of mass every 3 hours. 2

ii) What is the mass of the yeast in grams put into the vat at the beginning of the fermentation? 2

iii) At what rate is the yeast increasing when there is 12,000 kg of yeast in the tank? 1

End of Question 14

Question 15 (15 marks) Commence on a NEW page. Marks

(a) A function is defined:

$$f(x) = \begin{cases} \tan x\,, & 0 < x \le \dfrac{\pi}{4} \\[2ex] 1, & \dfrac{\pi}{4} < x < \dfrac{3\pi}{4} \\[2ex] -\tan x, & \dfrac{3\pi}{4} \le x \le \pi \end{cases}$$

i) Sketch the graph of $y = f(x)$. 1

ii) Show that 1

$$\int \tan x \; dx \; = -\log_e(\cos x) + C$$

iii) Find the area bounded by $y = f(x)$, the x-axis, $x = 0$ and $x = \pi$. 4

iv) The curve $y = f(x)$ is rotated about the x-axis. 4

Find the volume of the solid of revolution between $x = 0$ and $x = \pi$.

(b) The acceleration of a particle travelling along the x-axis is given by the equation

$$\ddot{x} \; = 6t - 18$$

where t is the time in seconds, and the acceleration is measured in m/s^2.

The particle has an initial velocity of 15 m/s.

i) Find the velocity at time t. 2

iii) At what times does the particle change direction? 1

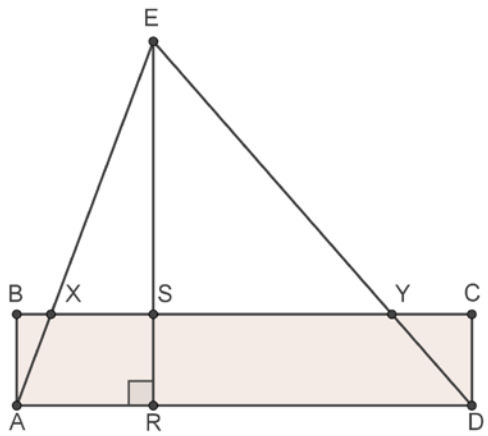
ii) What is the total distance travelled in the first 2 seconds? 2

End of Question 15

Question 16 (15 marks) Commence on a NEW page.

(a) A triangle AED is constructed using the base of a rectangle ABCD, with intersection points X and Y as shown. ER is the altitude of the triangle AED.

The area of triangle AED is twice the area of the rectangle.



i) Explain why $ER : ES = 4:3$ **2**

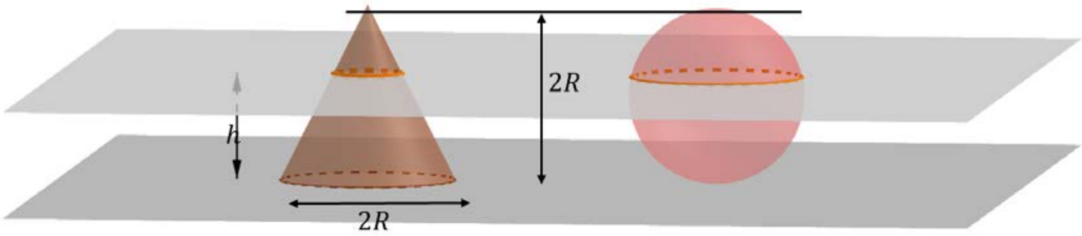
ii) Prove that $\Delta AED \parallel\!| \, \Delta XEY$. **3**

iii) Hence or otherwise, show that $4BX + 4YC = AD$. **3**

Question 16 continues on the next page

b) A sphere of radius R and a right circular cone with radius R at the base and height $2R$ are sitting on a horizontal plane.

A second horizontal plane, height h above the first plane, slices through the sphere and the cone, creating two circular cross sections in the sphere and the cone.



Let the radius of the cross-section in the sphere be r_s and the radius of the cross-section in the cone, r_c.

i) Show that for cross section of the sphere, $r_s^2 = 2Rh - h^2$ 2

ii) Show that for the cross section of the cone $r_c^2 = \left(R - \frac{h}{2}\right)^2$ 2

iii) Hence or otherwise find the height of the slicing plane which gives the maximum sum of cross-sectional areas. 3

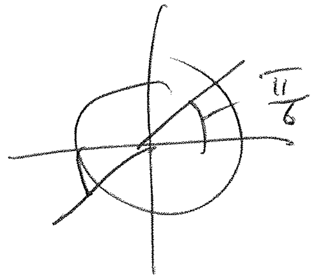
End of paper.

Section 1. Multiple Choice.

Q1. $\csc\frac{7\pi}{6} = \frac{1}{\sin\frac{7\pi}{6}}$

$= -\frac{1}{1/2}$

$= -2$ (A)



Q2. $\sum_{i=10}^{21} 2i-30$

$n = (21-10)+1 = 12$ terms

first $= 2(10)-30 = -10$

last $= 2(21)-30 = 12$

$S_n = \frac{12}{2}(-10+12)$

$= \frac{12}{2}\times 2$

$= 12$ (D)

Q3. $3x + 4y + 7 = 0$

perp. line is

$4x - 3y + c = 0$

$8x - 6y + 2c = 0$ (C)

Q4. $h = \frac{3-1}{4} = 1/4$

Area $\approx \frac{1}{4}(1f_0 + 2f_1 + 2f_2 + 2f_3 + 2f_4 + f_5)$

$\frac{1}{4}(1\times 3\log 3\ldots)$ — $\frac{1}{4}(1\log_e 1 + 2\times 1.5\log_e 1.5 + 2\times 2\log_e 2 + 2\times 2.5\log_e 2.5 + 1\times 3\log 3)$

(B)

Q1. A

Q2. D.

Q3. C.

Q4. B.

Q5. B.

Q6. D.

Q7. C.

Q8. D.

Q9. C

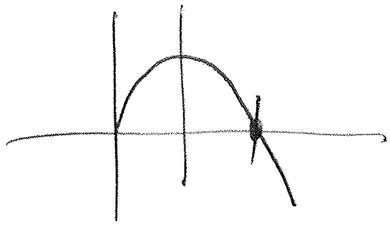
Q10. B.

Q5. $\lim_{x \to 0} \frac{\sin x}{x} = 1$ in radians.

$x \text{ radians} = \frac{\pi}{180} x \text{ degrees}$

$\lim_{x \to 0} \frac{\pi}{180}\left(\frac{\sin x}{x}\right) = 0.0174$ (B)

Q6. $\frac{dP}{dt} = 40 \sin(0.1 t)$



$T = \text{Period} = \frac{2\pi}{0.1} = 62.83$

$40 \sin 0.1 t < 0$

when $t = T/2 = 31.41$ (D)

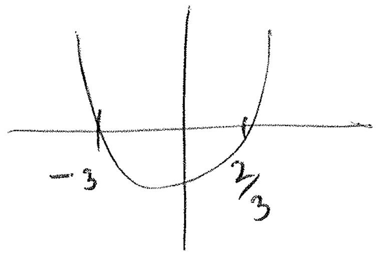
after day $\underline{31}$ - so day $\underline{32}$

[Day 1: $0 < t < 1$]

Q7. $3x^2 + 7x - 6 = 0$

$(3x - 2)(x + 3) = 0$

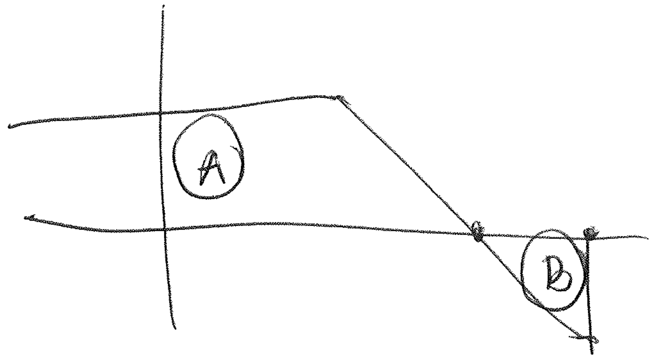
$x = 2/3 \quad x = -3$



(C)

so: $x < -3$ or $x > 2/3$

Q8

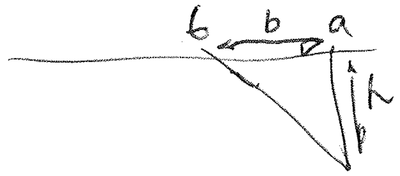


Area (A) is a trapezium $= \frac{1}{2}(8+12) \times 4 = 10 \times 4 = 40 u^2$

want (A) : (B) $= 8$

$\therefore$ Area B $= 32 \ u^2$

Now



$\therefore \quad \frac{1}{2} \cdot b \cdot h = 32$

But gradient $= 1$, $\therefore b = h$

$\therefore \quad \frac{1}{2} b^2 = 32$

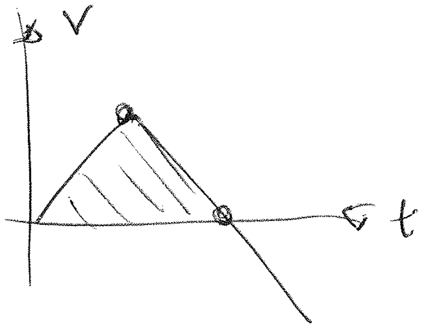
$b^2 = 64$

$b = 8$.

$a = 6 + 8 = \underline{14}$

$\therefore$ Answer (D)

Q.9.



Max value x - when $v=0$ (turns around)

$x = \int v\,dt + C$

$= \text{area triangle} + C$

$= \frac{1}{2}(10)(5) + C$

$= 25 + C$

Now $x(0) = 5$

$\therefore x_{max} = 25 + 5$

$= 30$ m. (C)

Q10.

$y = x^x$

$= \left(e^{\ln x}\right)^x$

$= e^{(x \ln x)}$

$y' = e^{(x \ln x)} \cdot \frac{d}{dx}(x \ln x)$

$= e^{x \ln x} \cdot (1 \times \ln x + x \times \frac{1}{x})$

$= (1 + \ln x) e^{x \ln x}$

$= (1 + \ln x) x^x$ (B)

Section II.

Q11. (a) $\dfrac{1-\sqrt{3}}{5-\sqrt{3}} = \dfrac{1-\sqrt{3}}{5-\sqrt{3}} \times \dfrac{5+\sqrt{3}}{5+\sqrt{3}}$

$$= \frac{(1-\sqrt{3})(5+\sqrt{3})}{25-3}$$

$$= \frac{5+\sqrt{3}-5\sqrt{3}-3}{22}$$

$$= \frac{2-4\sqrt{3}}{22}$$

$$= \frac{1-2\sqrt{3}}{11}$$

(b) $\lim\limits_{x \to 3} \dfrac{x^3-27}{x^2-9} = \lim\limits_{x \to 3} \dfrac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}(x+3)}$

$$= \lim_{x \to 3} \frac{x^2+3x+9}{x+3}$$

$$= \frac{(3)^2+3(3)+2}{3+3}$$

$$= \frac{27}{6}$$

$$= 9/2$$

Must show working for 2 marks.
question said "derive"

Q11.

(c) $|2x+1| > 6$

$2x+1 > 6$ or $-(2x+1) > 6$

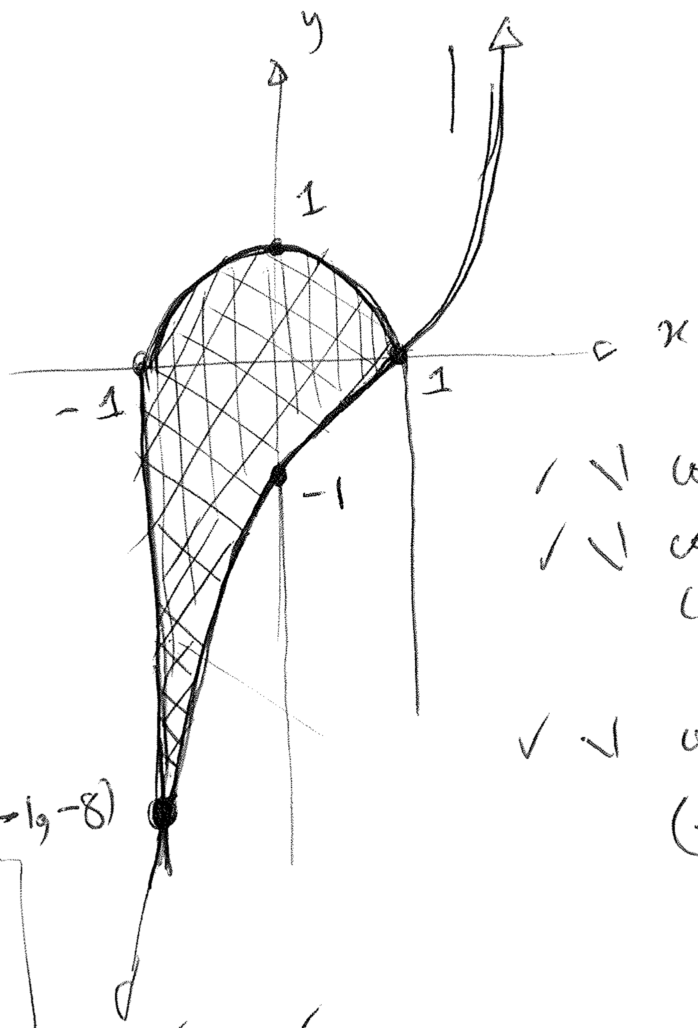
$2x+1 < -6$

$2x > 5$ $\quad$ $2x < -7$

$x > 5/2$ or $x < -\frac{7}{2}$ $\quad$ ✓1 per solution ✓✓

(d) $y \geq (x-1)^3$

$y \leq \sqrt{1-x^2}$



✓ ✓1 correct semi circle

✓ ✓1 correct cubic (with intercept x-y)

✓ ✓1 correct region

(-1, -8) not required.

(e) $y = 4x^3 - \sqrt{x}$

$= 4x^3 - x^{\frac{1}{2}}$

$y' = 12x^2 - \frac{1}{2}x^{-\frac{1}{2}}$

$= 12x^2 - \frac{1}{2\sqrt{x}}$ ✓✓

12 marks - deduct 1 per error.

(f) $\int\left(x^3 - \frac{2}{x}\right)dx = \frac{1}{3}x^2 - 2\log_e x + C$ ✓ ✓

12 marks

Deduct one per error.

Must have $\underline{+C}$ for full marks

(g) $\int_0^\pi \sin 2t \, dt = \left[-\frac{1}{2}\cos 2t\right]_0^\pi$ ✓1

$= -\frac{1}{2}(\cos 2\pi - \cos 0)$

$= 0$ ✓1

Q12. $A(-4,-1)$, $B(6,1)$, $C(0,5)$

(i) $x - 5y - 1 = 0$

Point A : $(-4) - 5(-1) - 1 = -4 + 5 - 1 = 0$ $\therefore$ A on the line.

B : $(6) - 5(1) - 1 = 6 - 5 - 1 = 0$ $\therefore$ B on the line ✓ 1

(ii) Distance AB :

$$d^2 = (6+4)^2 + (2)^2$$
$$= 100 + 4$$
$$= 104$$

$AB = \sqrt{104}$ units. ✓ 1

$= 2\sqrt{26}$. ← (optional)

(iii) Perp height AB to C is   AB : $x - 5y - 1 = 0$
C : $(0, 5)$

$$d = \frac{|(0) - 5(5) - 1|}{1^2 + 5^2}$$ ✓ using perp. distance.

✓ correct value.

$$= \frac{26}{\sqrt{26}}$$

$\therefore$ Area $\triangle ABC = \frac{1}{2}(2\sqrt{26}) \times \frac{26}{\sqrt{26}}$ ✓ using (ii) with perp.

$= 26$ units^2

Q12 (b)

(i) $y = \cos(7 - x^4)$

$y' = -\sin(7-x^4)\cdot(-4x^3)$

$= 4x^3 \sin(7-x^4)$ ✓✓

(ii) $y = \log\left(\frac{2x+1}{x-1}\right)$

$= \log(2x+1) - \log(x-1)$ ✓ Easy way

$y' = \frac{1}{2x+1} - \frac{1}{x-1}$ ✓

OR $y' = \frac{(x-1)}{2x+1} \times \frac{2(x-1)-(2x+1)}{(x-1)^2}$ Hard way - chain rule, quotient rule.

$= \frac{(x-1)}{(2x+1)} \times \frac{-3}{(x-1)^2}$

$= \frac{-3}{(2x+1)(x-1)}$

(c) (i) $\int 3e^{-5x}\,dx = -\frac{3}{5}e^{-5x} + C$

(ii) $\int x^2(1-\sqrt{x})\,dx = \int (x^2 - x^2\sqrt{x})\,dx$ ✓ for expanding

$= \int (x^2 - x^{5/2})\,dx$

$= \frac{1}{3}x^3 - \frac{2}{7}x^{7/2} + C$ ✓ correct answer.

(iii) $\int \frac{6x}{x^2-1}\,dx = 3\log_e(x^2-1) + C$ ✓✓

Q13

(a) $y^2 + 8x - 2y + 25 = 0$

$y^2 - 2y = -8x - 25$

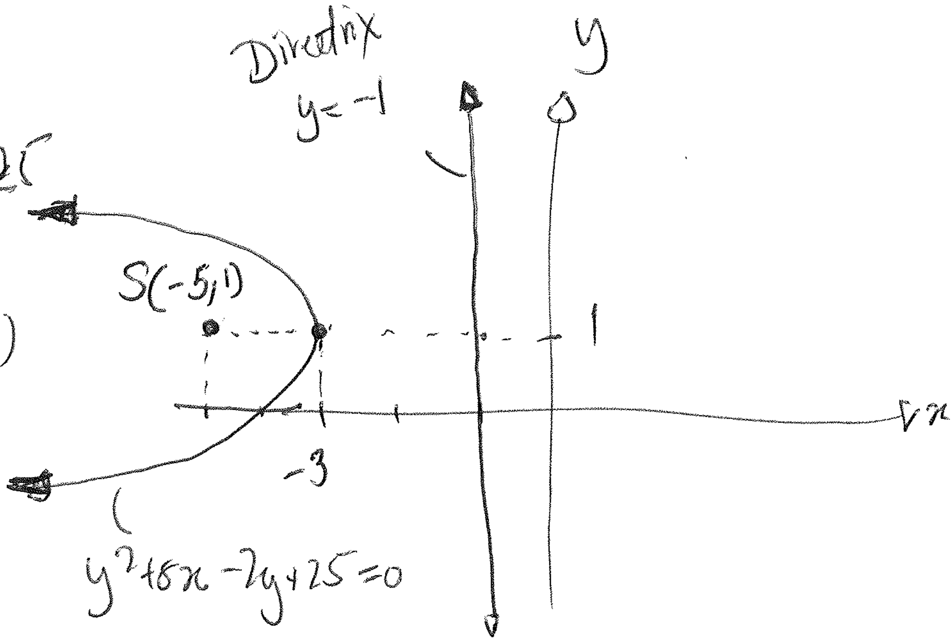
✓ complete square correctly $(y-1)^2 - 1 = -8x - 25$

$(y-1)^2 = -8x - 24$

$(y-1)^2 = -8(x+3)$

Vertex $(-3, 1)$

$a = 2$



✓ correct orientation

✓ correct vertex

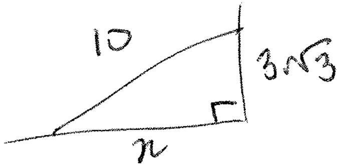
✓ correct focus and directrix

(b) $\dfrac{\sin 120^\circ}{10} = \dfrac{\sin C}{6}$

$\sin C = \dfrac{6 \times \sin 120^\circ}{10}$ ✓ correct use sine rule

$= \dfrac{3}{5} \times \dfrac{\sqrt{3}}{2}$

$= \dfrac{3\sqrt{3}}{10}$ ✓ correct value $\sin C$



$x^2 = 100 - 27$

$= 73$ ✓ find hypotenuse

$\therefore \tan C = \dfrac{3\sqrt{3}}{\sqrt{73}}$ ✓ correct value $\tan C$

(c) $y = 5x + k$ & $y = x^2 + 3$

Intersect where

$5x + k = x^2 + 3$

$x^2 - 5x + (3 - k) = 0$ ✓ Form equation

Require $\Delta \geq 0$ ✓ Use discriminant correctly

$\Delta = b^2 - 4ac$

$= 25 - 4(1)(3-k)$

$= 25 - 12 + 4k$

$= 13 + 4k$

$\therefore 13 + 4k \geq 0$

$k \geq -13/4$ ✓ Find value.

(d) $4\sin^2 x + 6\csc^2 x = 11$ $\quad 0 \leq x \leq 2\pi$

$4\sin^2 x + \dfrac{6}{\sin^2 x} = 11$

Let $u = \sin^2 x$

$4u + \dfrac{6}{u} = 11$

$4u^2 + 6 = 11u$

✓ getting quadratic $\rightarrow 4u^2 - 11u + 6 = 0$

$4u^2 - 8u - 3u + 6 = 0$

$4u(u-2) - 3(u-2) = 0$

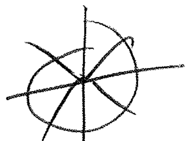
$(4u-3)(u-2)$

$\therefore 4\sin^2 x - 3 = 0$ or $\sin^2 x - 2 = 0$

Not possible ✓ for rejecting

$\therefore \sin^2 x = 3/4$

$\sin x = \pm\dfrac{\sqrt{3}}{2}$



$x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

✓✓ 1 mark per pair of solutions - (some will forget $\pm\sqrt{3}/2$)

Q14.

(a) $f(x) = 6x^3 - x^4$

(i) $6x^3 - x^4 = 0$

$x^3(6 - x) = 0$

x intercepts at $(0,0)$, $(6,0)$ ✓

(ii) $f'(x) = 18x^2 - 4x^3$

$= 2x^2(9\cancel{x} - 2x)$

$\therefore f'(x) = 0$ at $x = 0$, $x = 9/2$

$(0,0)$ $\left(9/2, \frac{2187}{16}\right)$ ✓✓ find two stationary points

$f''(x) = 36x - 12x^2$

x	0	$9/2$	
$f'(x)$	0	0	
$f''(x)$	0	-81	

can't tell.

$\boxed{\text{local max at } \left(9/2, \frac{2187}{16}\right)}$ ✓✓ determine nature.

x	-1	0	1
$f'(x)$	22	0	14
	/	—	/

$\boxed{\text{horizontal point of inflexion at } (0,0)}$

(iii) $f''(x) = 36x - 12x^2$

$= 12x(3-x)$

Possible inflexion at $x=0, x=3$

already shown inflexion

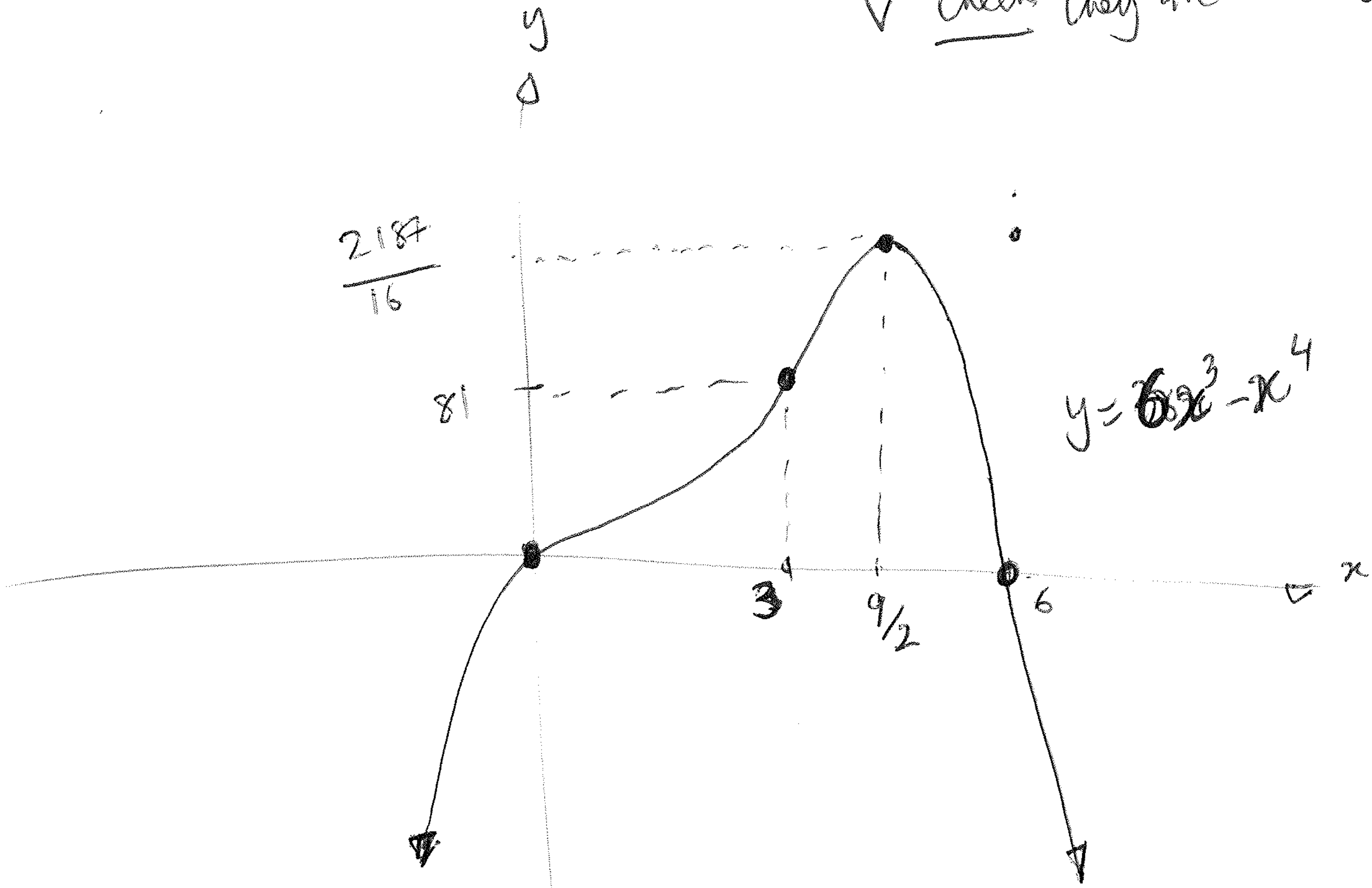
x	$\cancel{1}$	3	4
$f''(x)$	24	0	-48
	$\cup$		$\cap$

changes concavity

$\therefore$ Inflexion points at $(0,0), (3,81)$

✓ find points of inflexion

✓ check they are inflexions!



✓ general shape passing through x-intercepts

✓ correct local max shown

✓ inflexions clearly showing at $(0,0), \left(\frac{9}{2}, \frac{2187}{6}\right)$

Q14(b)

$P = P_0 e^{kt}$

(i) Double in 3 hours:

$2P_0 = P_0 e^{3k}$

$2 = e^{3k}$ ✓

$\ln 2 = 3k$

$k = \frac{\ln 2}{3}$ ✓

2.5 days = 60 hrs

(ii) $30{,}000 = P_0 e^{\left(\frac{\ln 2}{3}\right)(60)}$ ✓ correct substitution

$P_0 = \frac{30{,}000}{e^{20\ln 2}}$

≈ 0.0286 kg ✓ correct answer in grams.

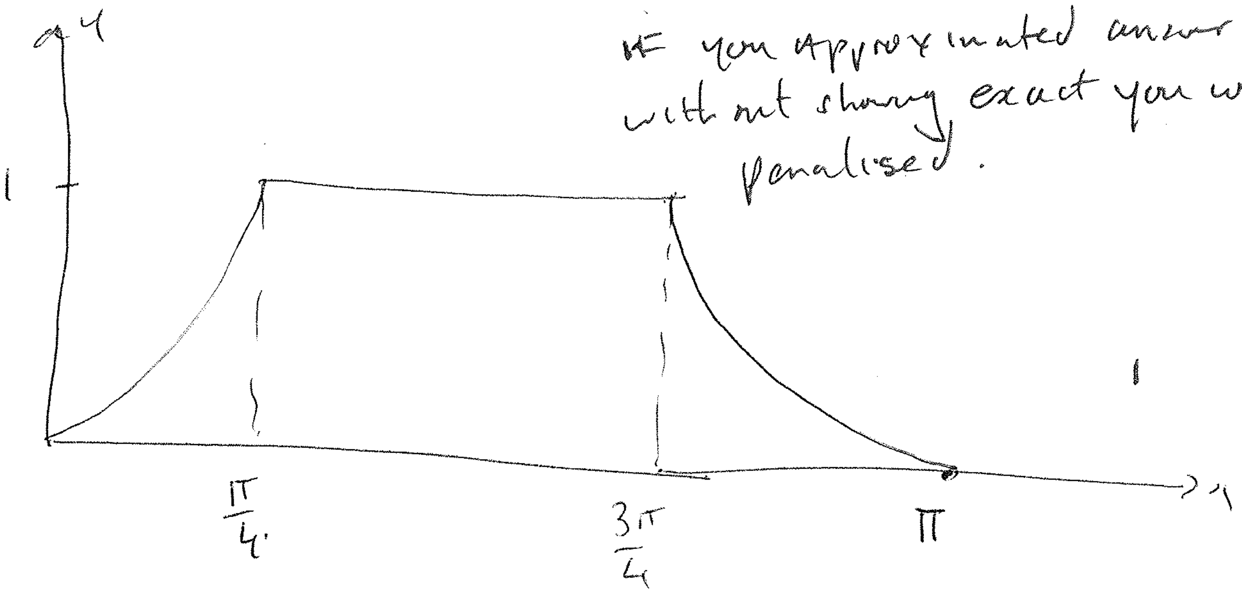
≈ 28.6 gms.

(iii) $\frac{dP}{dt} = k \cdot P$

$= \frac{\ln 2}{3} \cdot 12{,}000$

≈ 2773 kg/hr ✓

15a) i)



If you approximated answer without showing exact you were penalised.

1

~~1~~
~~2~~
~~3~~
~~4~~
~~5~~
~~6~~
~~7~~
~~8~~

ii) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\ln(\cos x) + c$ 1

iii) $A = 2\int_0^{\pi/4} \tan x \, dx + \frac{\pi}{2} \times 1$ 2

$= 2\left[-\ln(\cos x)\right]_0^{\pi/4} + \frac{\pi}{2}$ 1

$= 2\left(-\ln \frac{1}{\sqrt{2}} - (-\ln 1)\right) + \frac{\pi}{2}$ 1

$= 2\ln\sqrt{2} + \frac{\pi}{2} = \ln 2 + \frac{\pi}{2} \; u^2$

$\tan^2 + 1 = \sec^2$

$\tan^2 = \sec^2 - 1$

iv) $V = \pi\int_0^{\pi} y^2 \, dx = 2\pi\int_0^{\pi/4} \tan^2 x \, dx + \pi\int_{\pi/4}^{3\pi/4} 1 \, dx + \pi\int_{3\pi/4}^{\pi} \tan^2 x \, dx$ 1

$= 2\pi\int_0^{\pi/4} (\sec^2 x - 1) \, dx + \pi\left[y\right]_{\pi/4}^{3\pi/4} + \pi\int_{3\pi/4}^{\pi} (\sec^2 x - 1) \, dx$ 1

$= 2\pi\left[\tan x - x\right]_0^{\pi/4} + \pi\left[\frac{3\pi}{4} - \frac{\pi}{4}\right] + \pi\left[\tan x - x\right]_{3\pi/4}^{\pi}$ 1

$= \pi\left[1 - \pi/4\right] + \frac{\pi^2}{2} + \pi\left[(0 - \pi) - (-1 - 3\pi/4)\right]$

$= \pi - \frac{\pi^2}{4} + \frac{\pi^2}{2} + \pi\left[-\pi + 1 + \frac{3\pi}{4}\right]$

$= \pi - \frac{\pi^2}{4} + \frac{\pi^2}{2} + \pi - \frac{\pi^2}{4}$

$= \underline{\underline{2\pi \; u^3}}$ 1

b) $\ddot{x} = 6t - 18$ Q15 (b)

i) $\dot{x} = 3t^2 - 18t + c$

$t = 0 \quad \dot{x} = 15$

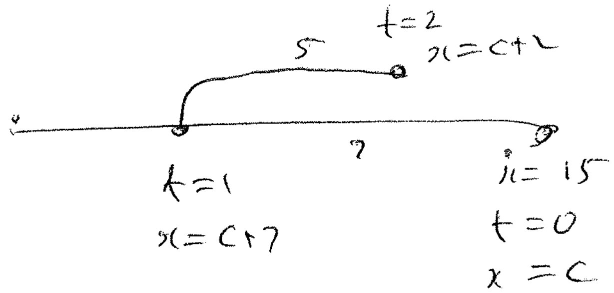
$c = 15$ (1 if no 15)

$\dot{x} = 3t^2 - 18t + 15$ 2

iii) $\dot{x} = 0 \quad t^2 - 6t + 5 = 0$

$(t-5)(t-1) = 0$

$t = 1$ or $t = 5$ changes direct at $t = 1$ + $t = 5$ 1

ii) 

1 $x = t^3 - 9t^2 + 15t + c$

$t = 0 \quad x = c$

$t = 1 \quad x = 7 + c$

$t = 2 \quad x = 8 - 36 + 30 + c$

$= 2 + c$

$= c + 2$

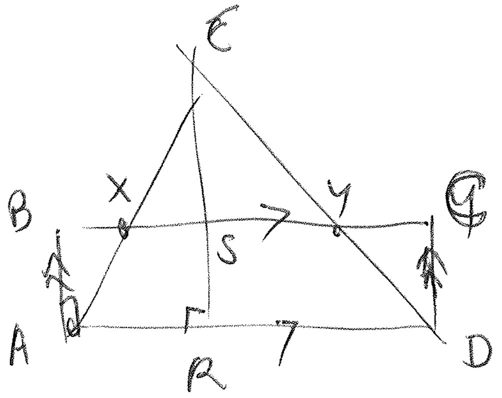
So Distance travelled $= 7 + 5$

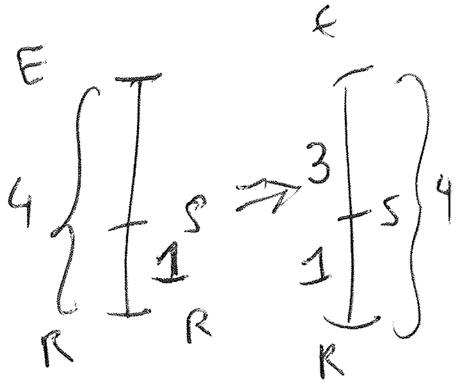
$= 12$ 1

integrate but
dont break up
Give 1

Q16.

(a)





Area $\triangle AED = 2 \times$ Area $ABCD$ ✓1

$\frac{1}{2}(AD)(ER) = 2 \times AD \times SR$

$\cancel{AD}\ ER = 4SR$

$ER = 4(ER - ES)$

$ER = 4ER - 4ES$

$4ES = 3ER$

$\frac{4}{3} = \frac{ER}{ES}$ ✓1

$\therefore ER : ES = 4 : 3$

sufficient to show $ER = 4SR$ then conclude $\frac{ER}{ES} = \frac{4}{3}$

(b) In $\triangle AED$ & $\triangle XEY$,

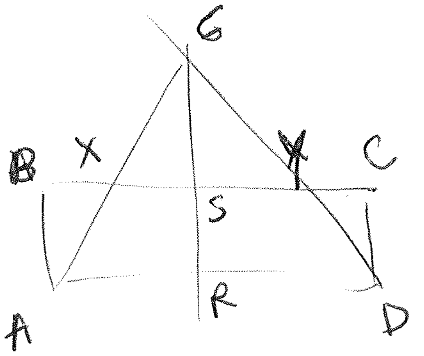
$\angle AED = \angle XEY$ (common) ✓1

$\angle EAD = \angle EXY$ ($BC \parallel AD$ – opp sides rectangle (alternate angles in parallel lines BC & AD are equal) ✓1

$\therefore \triangle AED \ ||| \ \triangle XEY$ (equiangular) ✓1

Q16(a)

(iii)



Since $\Delta AED \;|||\; \Delta XEY$,

matching sides are in proportion,

$\therefore \dfrac{XY}{AD} = \dfrac{3}{4}$ (from $\dfrac{ES}{ER} = \dfrac{3}{4}$)

✓ using sim. triangle ratio

$\therefore \dfrac{AD - BX - CY}{AD} = \dfrac{3}{4}$ ($BC = AD$ opposite sides rectangle.

$4(AD - BX - CY) = 3AD$

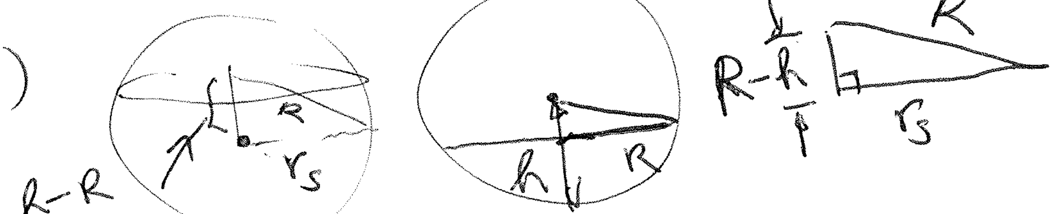
$4AD - 3AD = 4BX + 4YC$

$AD = 4BX + 4YC$ as required

✓ for working
✓ for conclusion

Q16(b)

(i)

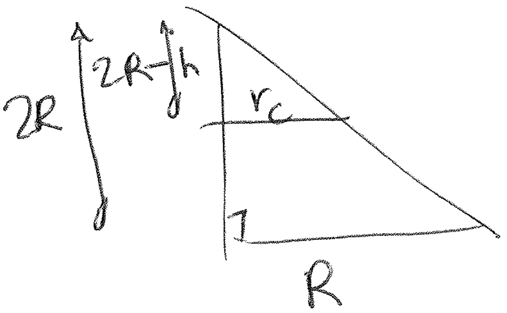


Using Pythagoras $R^2 = (R-h)^2 + r_s^2$ ✓

$r_s^2 = R^2 - (R^2 - 2Rh + h^2)$ ✓

$r_s^2 = 2Rh - h^2$ as required

Q16. (b) (ii)



similar triangles (equiangular)

$$\therefore \frac{r_c}{R} = \frac{2R-h}{2R} \quad ✓$$

$$r_c = R - \frac{h}{2} \quad ✓$$

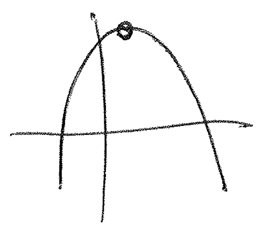
$$r_c^2 = \left(R - \frac{h}{2}\right)^2 \quad ✓$$

(iii) $\text{Area total} = \pi\left(r_s^2 + r_c^2\right)$

$$= \pi\left(2Rh - h^2 + \left(R - \frac{h}{2}\right)^2\right)$$

$$= \pi\left(2Rh - h^2 + R^2 - Rh + \frac{h^2}{4}\right)$$

$$= \pi\underbrace{\left(R^2 - \frac{3h^2}{4} + Rh\right)}_{\text{quadratic in } h,\ h < 0} \quad ✓$$



dA $= \cancel{2\pi}$

$= \cancel{A2}$

Maximum at $x = \frac{-b}{2a} \Rightarrow h = \frac{-R}{2\left(-3/4\right)}$ ✓ − justify maximum by graph or ~~first~~ derivative

$$\boxed{h = \frac{2}{3}R} \quad ✓$$

or: $\frac{dA}{dh} = \pi\left(-\frac{6}{4}h + R\right)$

$\frac{dA}{dh} = 0$ when $h = \frac{4}{6}R = \frac{2}{3}R$

$\frac{d^2A}{dh^2} = -\frac{2\pi 6}{4} < 0$ always $\therefore$ maximum value.